**Homework 2**

Michael Daniels

ISyE 6420

October 1, 2023

# Question 1

a)

f(x|Θ) ~ Posis(Θ)

π(Θ) = 1/√Θ

π(Θ|x) proportional to f(x|Θ)\*π(Θ)

π(Θ|x) = e-4Θ\*ΘΣx-(1/2) – note this is of similar form to gamma distribution alpha = 4.5, beta = 4

ΘMLE = Σx/n = (1+2+0+2)/4 = 1.25

Bayes estimator = mean of Gamma(5.5,4) = 5.5/4 = 1.375

Note the Bayes estimator is greater than the MLE.

b)

MatLAB code attached in appendix.   
95% Equitailed Credible Set for Accident Rate: (0.48, 2.74)

c)

95% HPD Credible Set for Accident Rate: (0.48, 2.72)

d)

Gamma Mode = (alpha-1)/beta = 5.5-1/4 = 1.125

e)

# Posterior Probability Accident Rate >= 1: 0.71

# Posterior Probability Accident Rate < 1: 0.29

# Do not reject H0.

# H0>H1. Evidence supports that the Accident Rate is >= 1

# Question 2

Find -E[

=

-E[ = -

da

Note that prior integral was a gaussian integral, I chose to emit the steps to expedite the process.

# Question 3

θi, i = 1, . . . , n (Note: If x ∼ Exp(θ),

then p(x) = 1/θe−x/θ and if x ∼ Inv−Gamma(α, β), then p(x) = 1/{βαΓ(α)}x−α−1e−1/(βx).).

Keeping this in mind.

Π(Θ|x) = f(x|Θ)\*π(Θ) = 1/Γ(α)\*Θi-α-1\*e-1/Θi\*(1/Θi)\*e-y/Θ = 1/Γ(α)\*Θ

We can say that the posterior π(Θ|x) is proportionate to Θi-α-2\*e-(yi+1/Θi). Which is an inverse gamma distribution around (α + 1, y+1).

The Bayesian estimator is (yi+1)/α

# Matlab Code

% Initialize observed accident data and prior information

data = [1, 2, 0, 2];

alpha\_prior = 0.5; % Starting alpha value from Jeffreys' prior

beta\_prior = 0; % No specific value given for Jeffreys' prior

% Update the parameters based on the data (resulting in a Gamma distribution)

alpha\_post = alpha\_prior + sum(data);

beta\_post = beta\_prior + length(data);

% Compute the 95% equitailed credible set for the accident rate

lower\_bound = gaminv(0.025, alpha\_post, 1/beta\_post);

upper\_bound = gaminv(0.975, alpha\_post, 1/beta\_post);

fprintf('95%% Equitailed Credible Set for Accident Rate: (%.2f, %.2f)\n', lower\_bound, upper\_bound);

% Determine the 95% HPD credible set using empirical samples from the posterior

n\_samples = 10000;

samples = gamrnd(alpha\_post, 1/beta\_post, [n\_samples, 1]);

samples = sort(samples);

hpd\_lower = samples(floor(0.025 \* n\_samples));

hpd\_upper = samples(floor(0.975 \* n\_samples));

fprintf('95%% HPD Credible Set for Accident Rate: (%.2f, %.2f)\n', hpd\_lower, hpd\_upper);

% Test hypotheses regarding the accident rate

posterior = @(theta) theta.^4.5 .\* exp(-4\*theta) .\* (theta > 0); % Combined prior and likelihood

% Normalize the posterior distribution using numerical integration

total\_area = integral(posterior, 0, Inf);

% Calculate posterior probabilities for both hypotheses

prob\_H0 = integral(posterior, 1, Inf) / total\_area;

prob\_H1 = 1 - prob\_H0;

fprintf('Posterior Probability Accident Rate >= 1: %.2f\n', prob\_H0);

fprintf('Posterior Probability Accident Rate < 1: %.2f\n', prob\_H1);

% Make a decision based on the higher posterior probability

if prob\_H0 > prob\_H1

disp('H0>H1. Evidence supports that the Accident Rate is >= 1');

else

disp('H0<H1. Evidence suggests the Accident Rate is < 1');

end